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Reply to Dr. Kuhn's discussion

The authors thank Dr. Kuhn for summarizing and discussing their paper. The main point of the discussion is that the postulate $\delta W_I^D = \delta W_I$ and $\delta W_E^D = \delta W_E$ is flawed because it implies non-uniqueness for (1) average stress (i.e. Eq. (44)) and (2) stress anti-symmetry (i.e. Eq. (47)), which can be obtained simply by shifting the reference points of peripheral particles.

At first, we point out that Eq. (44) yields a definition of average stress identical to those derived by independent investigators, e.g., Weber (1966), Goddard (1977), Christoffersen et al. (1981) and Rothenberg and Selvadurai (1981). The discussion therefore also affects not only our work but also that of others.

We agree with Dr. Kuhn that average stress may be defined non-uniquely, but disagree on the origin of this non-uniqueness. Non-uniqueness originates from the selection of reference points at which both granular and continuum media share identical displacements. Non-uniqueness does not originate from a problem with virtual work.

There are several ways to match the displacements of a granular media with that of a generalized continuum. We have assumed to match displacements at the particle centers (i.e. Eq. (27)). This is a physical assumption rooted in practical considerations. Real soil grains have very complicated shapes and geometries of quasi-random nature. As it is well known in Soil Mechanics, a complete geometrical description of all the individual grains within a soil mass is simply out of question in view of the very large number of grains. We have retained only the characteristics of grain geometries that are relevant for defining stress, and have assumed that soil particles can be viewed as spheres. A grain has volume V, which corresponds to the closest sphere to it with radius $R = \sqrt[3]{3V/4\pi}$. A sphere in turn is a very simple geometrical object; it has a center and all its points on the periphery are equidistant from it. Our assumption of selecting grain centers as reference point has definite physical and practical merits.

In conclusion, we reject Dr. Kuhn's claim that our postulate on virtual work is flawed because it leads to non-unique average stress and stress anti-symmetry. The non-uniqueness results from the assumption in Eq. (27), which is required for matching the displacements of structured granular media with those of amorphous continua. Additional research may be useful to bracket the variation in average stress arising from the assumptions in superimposing the kinematics of granular and continuum media.

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